EXAMS ROAD

"Bringing Excellence to Students"

Handwritten Notes

on

Kinetic Theory of Gases

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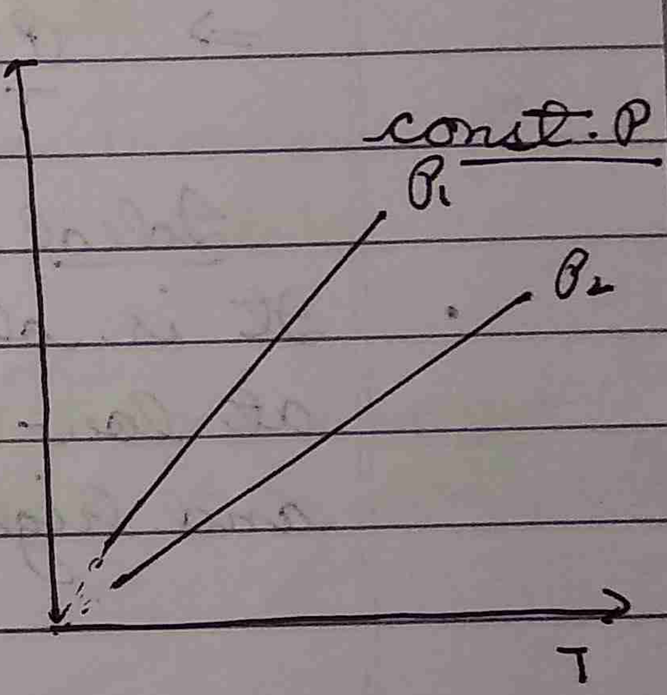
Gas Laws

1 Charle's law

At constant Pressure.

$V \propto T$ ← absolute temperature.

$$\frac{V}{T} = \text{const.} \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

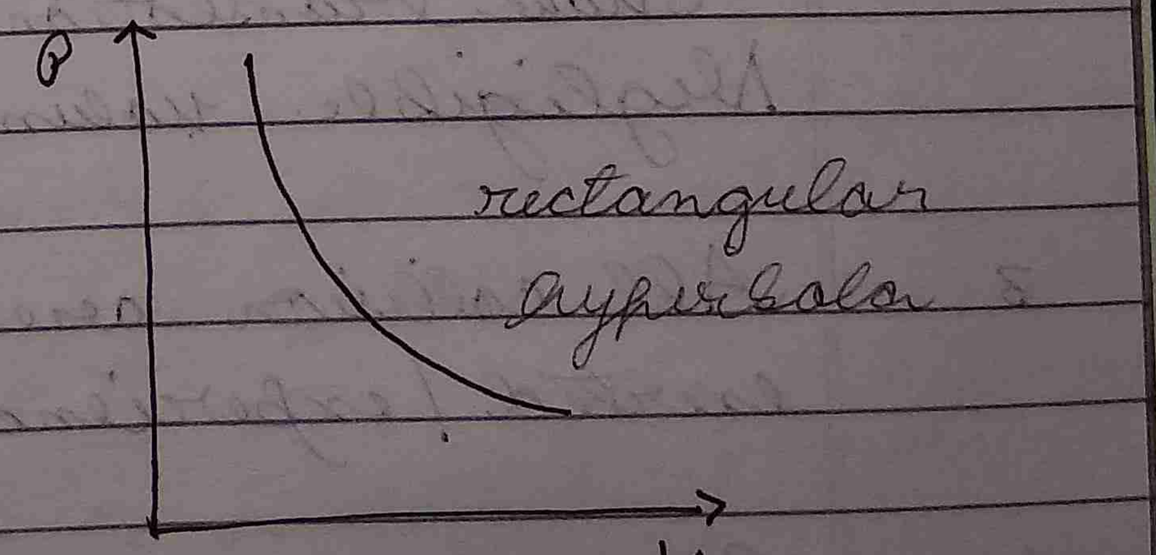


2 Boyle's law

At constant T

$P \propto \frac{1}{V}$ or $V \propto \frac{1}{P}$

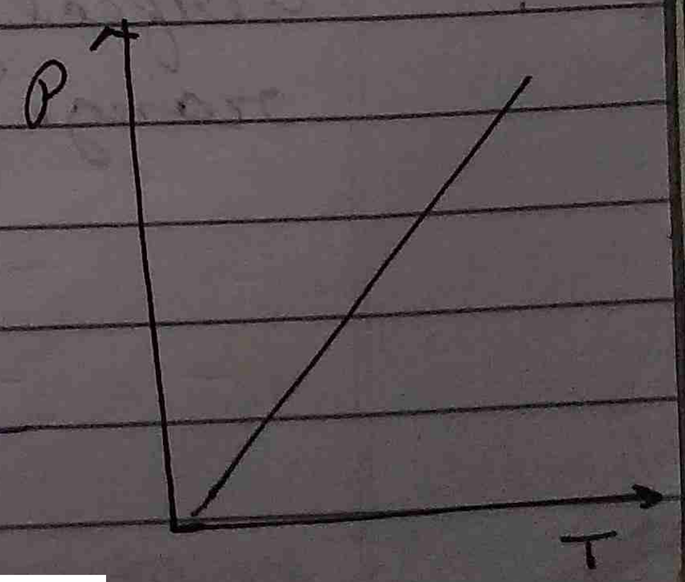
$PV = \text{const.}$



3 Gray-Lussac's law

At const. V

$P \propto T$ $\quad \frac{P}{T} = \text{const}$ $\quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$



☆ Hence U_P (Potential energy = 0)

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$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad P_1V_1 = P_2V_2$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{PV}{T} = \text{const} \Rightarrow PV = nRT$$

n → no. of moles.

R = Universal Gas constant = 8.314 J/mol K

Ideal gas equation

It is obeyed by gases at low pressure and high temperature.

Assumption

1. No intermolecular attraction or repulsion.☆
2. Molecules are tiny spheres (have translational motion only). Negligible volume occupied by gas molecule.
3. All collision are perfectly elastic, force exerted / experienced only during collision.
4. Perfectly random motion. range of velocity → zero to very large values

• Average velocity of gas in equilibrium is $\underline{0}$

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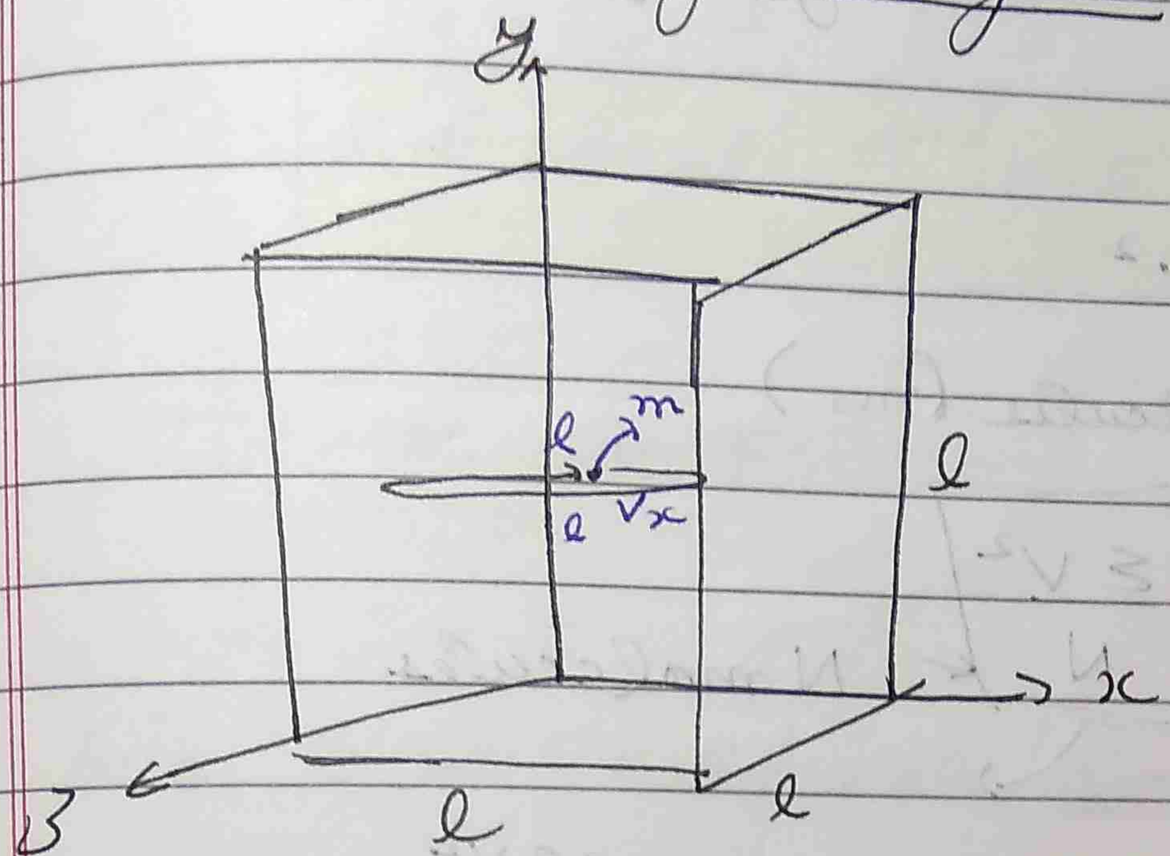
Pressure of a gas



• Time b/w two collisions $= \frac{2l}{V_x}$

• Collision frequency $= \frac{V_x}{2l}$ (per second)

• change in momentum/collision $= 2mV_x$

• change in momentum / sec $= 2mV_x \times \frac{V_x}{2l} = \frac{mV_x^2}{l}$

• If there are N molecules,

• Force in x direction (for a wall ∥ to Y–Z plane)

$$F_x = \frac{m}{l} \Sigma V_x^2 \Rightarrow F_x = \frac{m}{l} \Sigma V_x^2$$

• Pressure on the wall.

$$\begin{cases} P_x = \frac{F_x}{l^2} = \frac{M}{l^3} \Sigma V_x^2 \\ P_y = \frac{m}{l^3} \Sigma V_y^2 \\ P_z = \frac{m}{l^3} \Sigma V_z^2 \end{cases}$$

$$\star\ P_x = P_y = P_z = P$$

$$\Rightarrow P = \frac{P_x + P_y + P_z}{3}$$

$$P = \frac{m}{3l^3} \times \Sigma (v_x^2 + v_y^2 + v_z^2)$$

$$= \frac{m}{3l^3} \cdot \Sigma v^2$$

Total mass of gas molecules (m_0)

$$P = \frac{mN}{3V} \cdot \frac{\Sigma v^2}{N}$$

mN → Total mass of gas molecules

$3V$ → volume

$\frac{\Sigma v^2}{N}$ ← N molecules

average of v^2 (Mean sq. velocity)

$$= \frac{\rho}{3} (v_{rms})^2$$

$$P = \frac{1}{3} \rho v_{rms}^2$$

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

- Kinetic interpretation of temperature

$$P = \frac{1}{3} \rho v_{rms}^2$$

$$= \frac{1}{3} \frac{m_0}{V} v_{rms}^2 .$$

$$\Rightarrow PV = \frac{1}{3} m_0 v_{rms}^2$$

$$= \frac{2}{3} \times \underbrace{\frac{1}{2} m_0 v_{rms}^2}$$

↘ K.E of the gas molecules in the vessel.

$$PV = \frac{2}{3} K.E$$

But $PV = nRT$

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$\Rightarrow nRT = \frac{2}{3} K.E$

$\Rightarrow$ **$K.E/mole = \frac{K.E}{n} = \frac{3}{2} RT$**

$T \propto K.E$

- Avg. trans K.E/mole/direction $= \frac{1}{3}\left(\frac{3}{2}RT\right)$

$= \frac{1}{2}RT$

$\Rightarrow$ equipartition of energy

- $K.E/molecule = \frac{3}{2}RT / N_a = \frac{3}{2}K_B T$

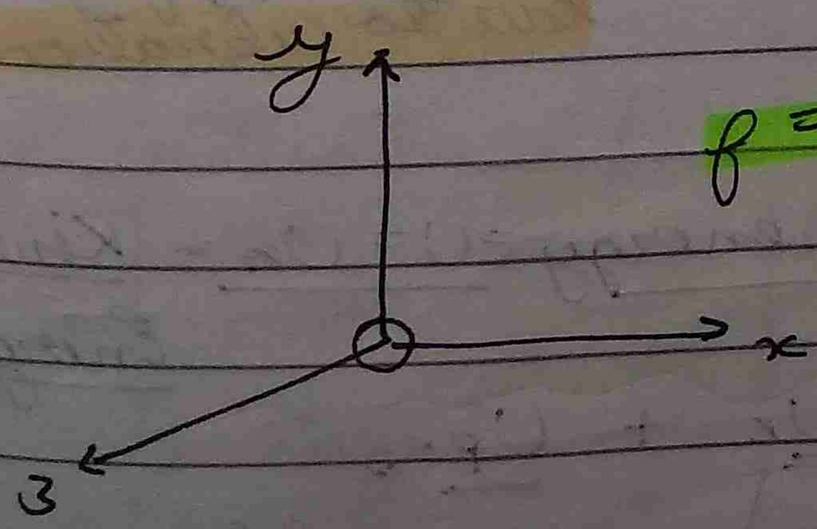
- $K.E/molecule/direction = \frac{1}{2}K_B T$

Boltzmans constant $= 1.38 \times 10^{-23}$ J/K

Degree of freedom (f)

No. of ways in which energy can be associated with a gas molecule.

Monoatomic



$f = 3$

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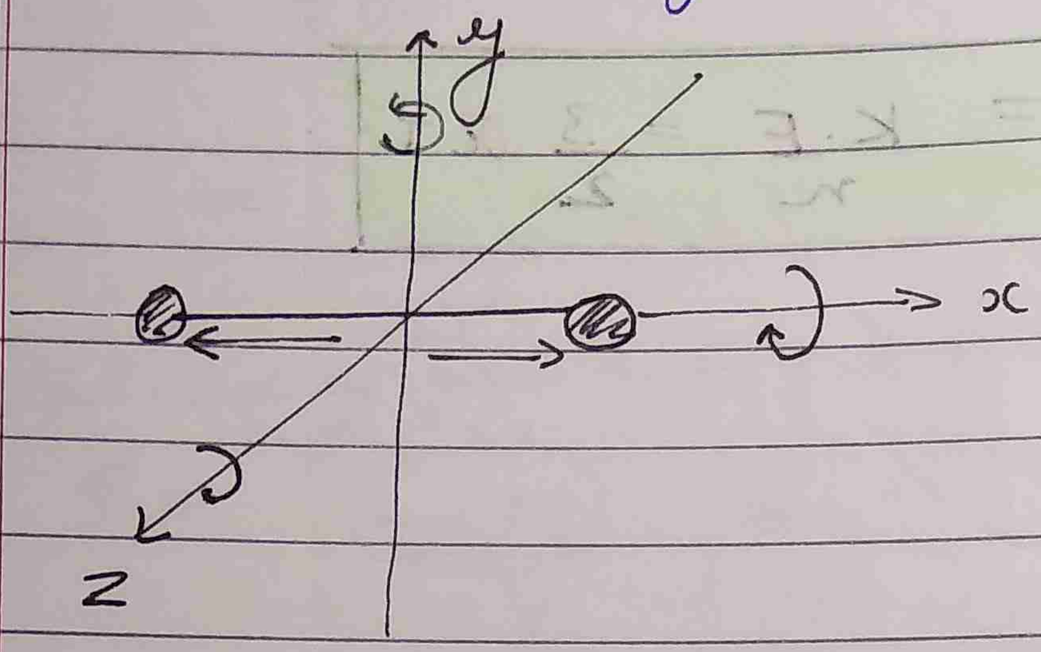
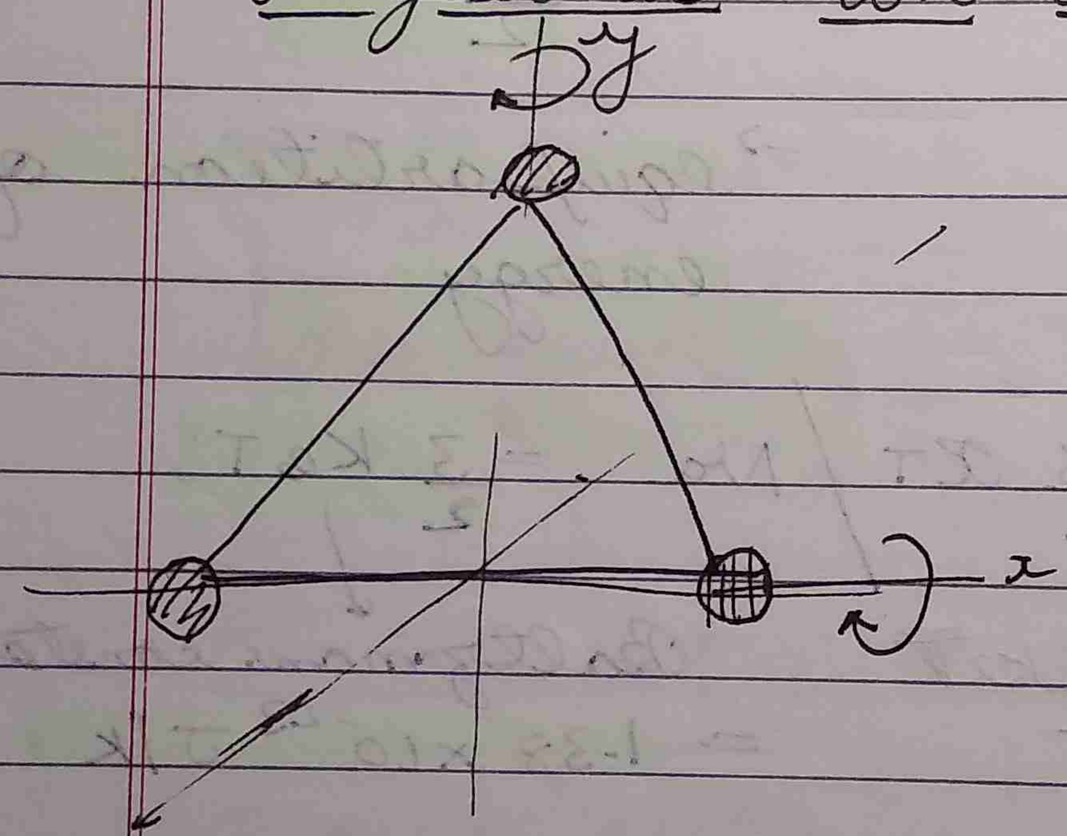
Diatomic / Polyatomic linear



f = 3 trans + 2 rot
= 5

Polyatomic non-linear molecule



f = 3 trans + 3 rot
= 6

Diatomic gas

< 70 K (3)

250 – 750 K (5)

> 1000 K (7) (+2 degree of freedom due to vibration)

Ideal gas → Internal energy = $U = U_K$ = Kinetic Energy

Real gas → $U = U_K + U_P$ →

$U = f(T)$

$U = f(T, V)$

depends on volume

K.E / mole / degree of freedom $= \frac{1}{2} RT$

- **Internal Energy of an ideal gas : per mole :-**

$$U = f \times \frac{1}{2} RT$$

$$U = n C_v T \Rightarrow C_v = \frac{U}{T}$$

$$\Rightarrow \boxed{C_v = \frac{1}{2} f R}$$

$$C_p = C_v + R = \frac{(f+2)}{2} \cdot R \qquad \boxed{C_p = \frac{(f+2)}{2} \cdot R}$$

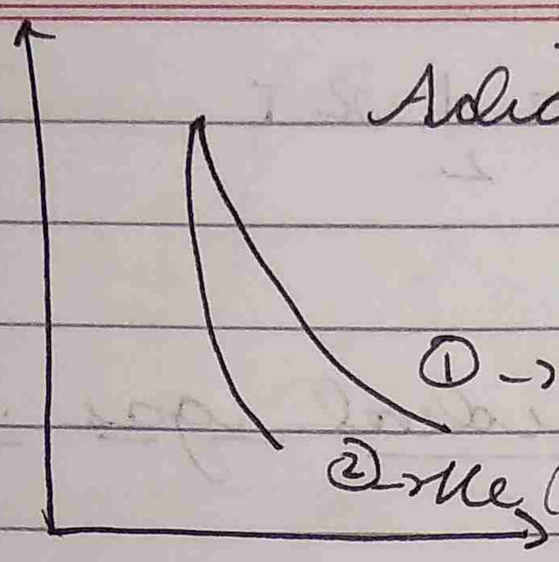
$$\gamma = \frac{C_p}{C_v} = \frac{f+2}{f} \Rightarrow \boxed{\gamma = 1 + \frac{2}{f}}$$

- **C_p, C_v and γ for diff. gases**

$\frac{f}{2} R \rightarrow \frac{f+2}{2} R \rightarrow 1 + \frac{2}{f}$

Type of gases	C_v	C_p	γ
1 Monoatomic	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3}$
2 Diatomic / Poly linear	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5}$
3 Poly non-linear	$3R$	$4R$	$\frac{4}{3}$

Q



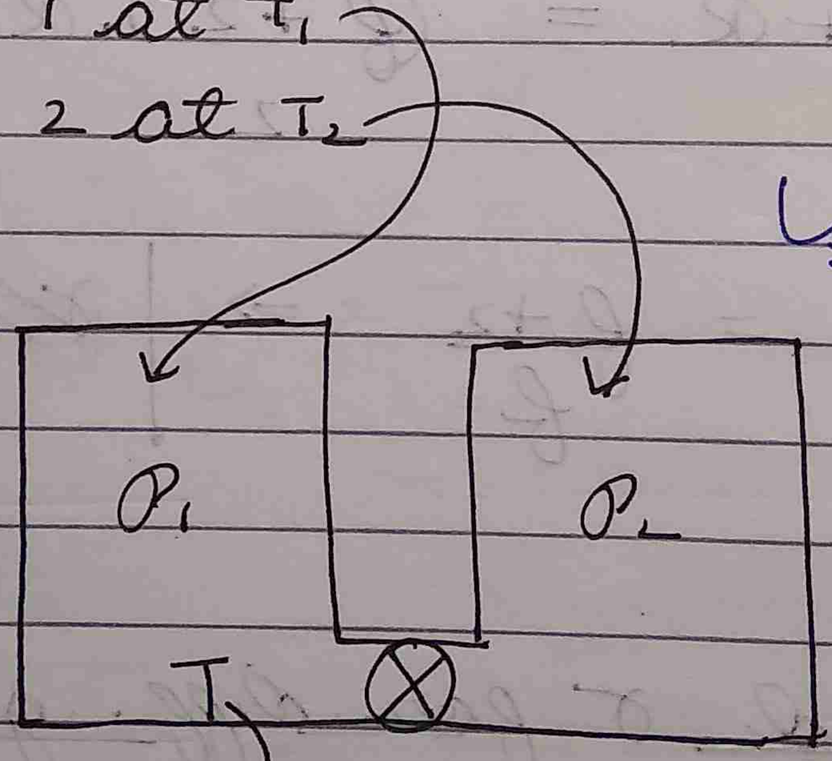
Adiabatic lines
He, O₂

slope = $-\gamma \frac{P}{V}$ of adiabatic line

Q. Mixture of gas $PV^{3/2}$ = constant

Mixture of gases

adiabatic vessel



$U_i = U_f$

$U_i = n_1 C_V T_1 + n_2 C_{V_2} T_2$

$U_f = (n_1 C_{V_1} + n_2 C_{V_2}) \cdot T$

$$T = \frac{n_1 C_{V_1} T_1 + n_2 C_{V_2} T_2}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$C_{v\,mix} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

$$C_{p\,mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2} \text{ or } C_{v\,mix} + R$$

$$\gamma_{mix} = \frac{C_{p\,mix}}{C_{v\,mix}}$$

Q $\underset{\rightarrow 300K}{1 \text{ mole of } O_2} + \underset{\rightarrow 400K}{2 \text{ mole } He}$

$$C_{v\,mix} = \frac{1 \times \frac{5}{2} R + 2 \times \frac{3}{2} R}{1+2} = \frac{11}{6} R$$

$$C_{p\,mix} = \frac{11R}{6} + R = \frac{17R}{6}$$

$$\gamma_{mix} = \frac{17R}{11}$$

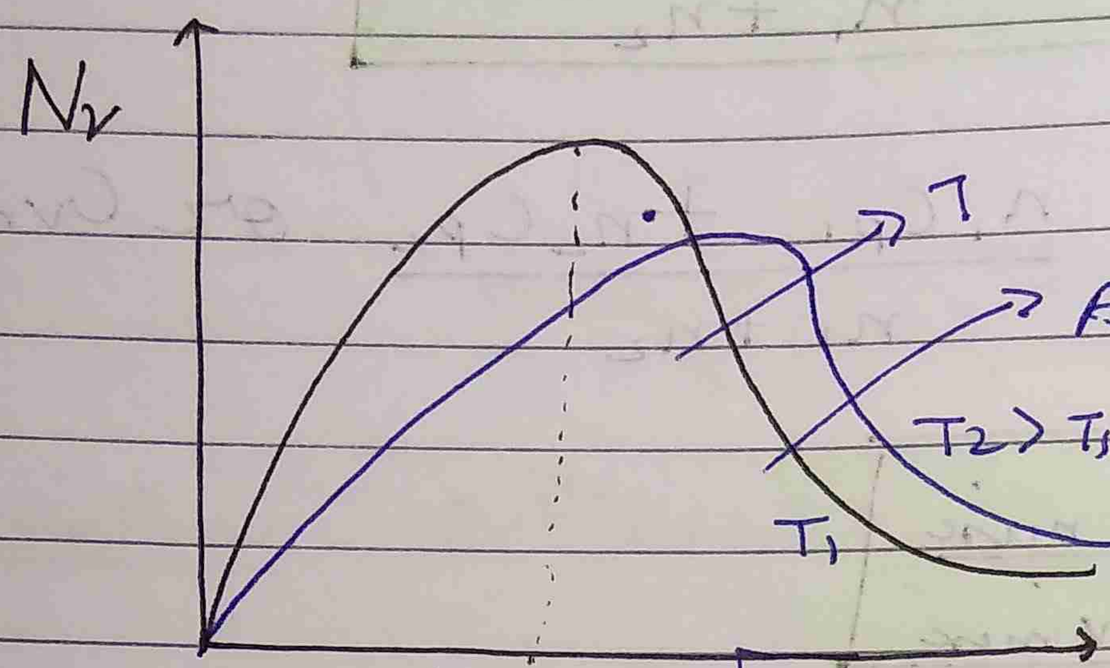
$$T_{mix} = \frac{1 \times \frac{5}{2} R \times 300 + 2 \times \frac{3}{2} R \times 400}{\frac{5}{2} R + \frac{6}{2} R} = 354.5\ K$$

☆ no of moles remains constant : $n = \frac{PV}{RT}$

(In a closed system)

☆ $$\frac{P(V_1 + V_2)}{RT} = n_1 + n_2 = \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}$$

Maxwell distribution of speeds.



→ Area ∝ N (Total no. of molecules of gas)

⇒ Asymmetric curve

N_v = Probability of molecule velocity b/w v and $v + dv$

dN = No. of molecules having speed b/w v and $v + dv$

$= N_v\, dv$

$$\boxed{dN = 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}\, dv}$$

☆ $\boxed{V_{av} = \sqrt{\frac{8K_BT}{\pi m}}}$ $\qquad$ $\boxed{V_{most\,p} = \sqrt{\frac{2K_BT}{m}}}$

☆ $V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{m}}$

$\rho \to \frac{PM}{RT}$

$V_{mp} < V_{av} < V_{rms}$

⇒☆ V of a gas is independent of pressure change it changes only when temperature changes.

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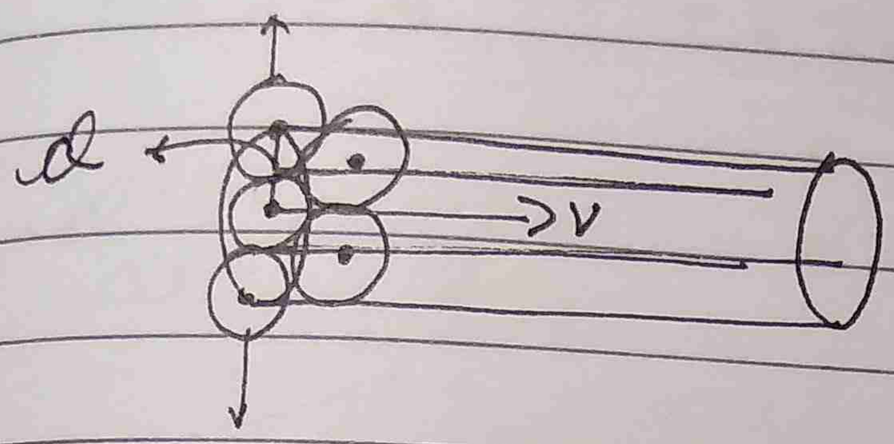
Mean free path λ



n = molecules / volume

$\bar{v}$ = average velocity of gas molecule.

$\pi d^2 \cdot \bar{v} t$ = volume of the cylinder in which gas molecule collides with other molecules.

Total no. of collisions $= n \times \pi d^2 \bar{v} t$

τ = Average time b/w two collisions

$$= \frac{t}{n \pi d^2 \bar{v} t} = \frac{1}{n \pi d^2 \bar{v}}$$

$$\lambda = \bar{v} \times t = \boxed{\lambda = \frac{1}{\sqrt{2} n \pi d^2}}$$

Characteristics of angular S.H.M

$\tau = -C\theta$ $\qquad \tau = I\alpha = I\frac{d^2\theta}{dt^2}$

$\frac{d^2\theta}{dt^2} = \frac{-C}{I}\theta$ i.e $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \frac{C}{I}$

$\frac{d\theta}{dt} = \omega\sqrt{\theta_0^2 - \theta^2}$ $\qquad \theta = \theta_0 \sin(\omega t + \phi)$