"Bringing Excellence to Students"

Handwritten Notes
on
Circular Motion

Circular Motion

① Angular displacement (θ)

2π rad $= 360°$

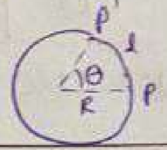
$\theta \rightarrow$ Axial Vector

Axial Vector $\rightarrow \theta, \omega, \alpha, \tau$

dirⁿ of $\theta \rightarrow$ by Right hand thumb rule

$\theta \rightarrow$ dimensionless

$\boxed{\theta = \frac{l}{R}}$



② Angular Velocity (ω)

Avg. Angular Vel.

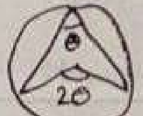
$\boxed{\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}}$

Instantenous angular vel.

$\boxed{\omega_{inst} = \frac{d\theta}{dt}}$

$\int d\theta = \int \omega dt$

③ Angle at Centre is twice angle at Circumference



④ $\omega \rightarrow$ Axial Vector

$\omega \rightarrow$ Rad/sec

$\boxed{\omega = \frac{d\theta}{dt}}$

$\theta = 2\pi$

$\omega = \frac{\theta}{T} = \frac{2\pi}{T}$

$\boxed{\omega = 2\pi\nu}$

$\nu \rightarrow$ frequency

$\boxed{\nu = \frac{\text{Revolution}}{\text{sec}} \quad \nu = \text{rpm.}}$

⑤ Angular accⁿ (α)

Avg. angular accⁿ

$\boxed{\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}}$

Instantenous angular accⁿ

$\boxed{\alpha_{inst} = \frac{d\omega}{dt}}$

$\omega = \int \alpha\, dt$

⑥

	Linear		Angular
(i)	S	$S = \theta R$	θ
(ii)	$V = \frac{ds}{dt}$	$V = \omega R$	$\omega = \frac{d\theta}{dt}$
(iii)	$a = \frac{dv}{dt}$	$a = \alpha R$	$\alpha = \frac{d\omega}{dt}$

⑦ For Constant / uniform angular accⁿ (α)

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

⑧

Angular	Linear
$\boxed{\theta = \left(\frac{\omega + \omega_0}{2}\right) \times t}$	$S = \left(\frac{V+U}{2}\right) \times t$
$\boxed{\theta^{th} = \omega_0 + \frac{\alpha}{2}(2t-1)}$	$S_{n^{th}} = U + \frac{a}{2}(2n-1)$

⑨

No. of Revolution (N) = $\dfrac{\text{Angular disp.}}{2\pi}$

$$\boxed{N = \frac{\theta}{2\pi}}$$

1 Rev. = 2π

Centripetal accⁿ (a_c)

(i) When particle is uniform circular motion, it Speed remains Constant
⇓
U.C.M

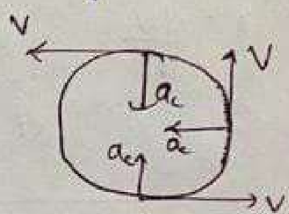
(ii) The Vel. of particle changes due to change in dirⁿ.

(iii) The accⁿ due to change in dirⁿ of Vel. is Centripetal accⁿ

(iv) Centripetal accⁿ is directed towards Centre of Circle.

$$\boxed{a_c = \frac{V^2}{R}}$$

Radial accⁿ



$$\boxed{a_c = \omega^2 R}$$

$V = \omega R$

only change in dirⁿ

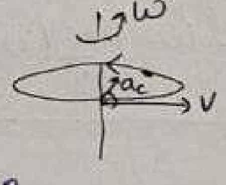
$$\boxed{a_c = V\omega}$$

Centripetal accⁿ also Called Radial accⁿ

a_R or a_c

Centripetal accⁿ is Constant for U.C.M
⇓
No. as its dirⁿ changes

$$\boxed{\vec{a}_c = \vec{\omega} \times \vec{V}}$$



$a_c = \omega V \sin 90$

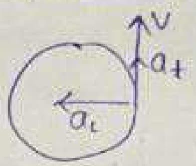
$a_c = \omega V$ $V = \omega R$

$$\boxed{a_c = \omega^2 R}$$

Tangential accⁿ (a_t)

(i) If Speed of particle is also changing in circular motion i.e Vel. also changes in Magnitude as well as dirⁿ
⇓
we have tangential accⁿ

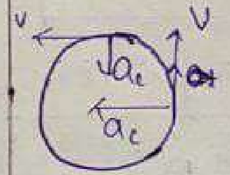
$$\boxed{a_t = \frac{d|V|}{dt}}$$

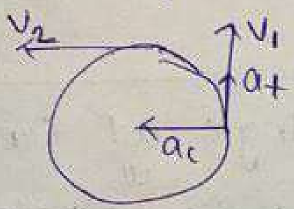


↳ act along tangent

UCM
⇓
only dir of $\vec{V}$ changes
⇓
a_c

NUCM
⇓
dir of $\vec{V}$ change → a_c
&
Magnitude also changes → a_t





Qn.) In U.C.M

a) $a_c \neq 0$ $a_t = 0$

⑩ Relⁿ b/w a_t & angular accⁿ (α)

$$a_t = \frac{d|V|}{dt} = \frac{d[R\omega]}{dt} = \frac{R\,d[\omega]}{dt}$$

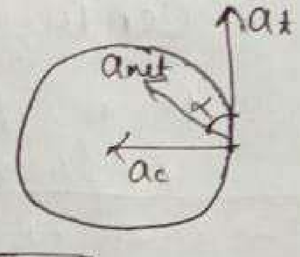
$$\boxed{a_t = R\alpha}$$

α tab hoga jab a_t hoga

$$a_c = \frac{V^2}{R} = \omega^2 R = \omega V$$

$$a_t = \frac{d|V|}{dt} = \alpha R$$

Net acc^n



$$\boxed{a_{net} = \sqrt{a_c^2 + a_t^2}}$$

$$\boxed{\tan\alpha = \frac{a_c}{a_t}}$$

For UCM → $\alpha = 0$

$a_t = 0$

$|V| \to$ Constant

$|\omega| \to$ Constant $\vec{V}$ = Constant ✗

$\vec{\omega}$ = Constant $a_c \neq 0$

$|a_c| \to$ Constant

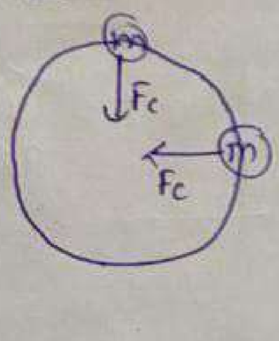
Dynamics of Circular Motion

Centripetal Force (F_c)

1) When particle moves in Circular path → Centripetal acc^n is present always.
2) This Centripetal acc^n a_c is directed along the radius of Circle towards Centre
3) As there is no acc^n without force ⇓
   A Centripetal force acts on particle along radius towards Centre.

$F = ma$

$$\boxed{F_c = ma_c}$$



$$\boxed{F_c = \frac{mv^2}{r}}$$

$$\boxed{F_c = m\omega^2 r}$$

$$\boxed{F_c = mv\omega}$$

$a_c = V\omega$

(ii) Work done by Centripetal Force (F_c) & Change in K.E

Work done in Round trip = 0

Work done in half trip = 0

$W = F \cdot ds$

$= F_s \cos\theta$

$W = 0$

Work Energy theorem

⇓

$$\boxed{\text{Work done} = \text{Change in K.E}}$$

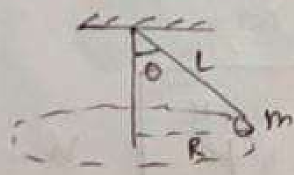
$\underset{0}{W} = K.E_f - K.E_i$

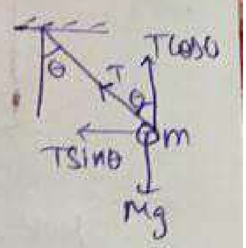
$$\boxed{K.E_i = K.E_f}$$

$$\boxed{V_i = V_f}$$

Speed Can Change if F tangential is present

Conical pendulum





$T\cos\theta = mg$ (i)

$T\sin\theta = F_c = \frac{mv^2}{R}$ (ii)

(ii) ÷ (i)

$$\tan\theta = \frac{v^2}{gr}$$

$$\boxed{V = \sqrt{gr\tan\theta}}$$

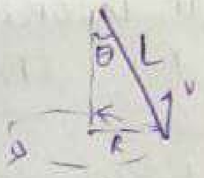
Time period $T = \frac{2\pi}{\omega}$

$\omega = \frac{V}{r}$ $T = \frac{2\pi}{\sqrt{\frac{g\tan\theta}{r}}}$

$$\boxed{T = 2\pi\sqrt{\frac{r}{g\tan\theta}}}$$

$$\sin\theta = \frac{r}{L}$$

$$r = L\sin\theta$$

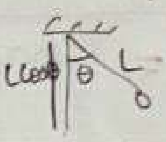


$$T = 2\pi\sqrt{\frac{L\sin\theta}{g\tan\theta}} \Rightarrow \frac{\sin\theta}{\cos\theta}$$

$$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Shortcut for conical pendulum

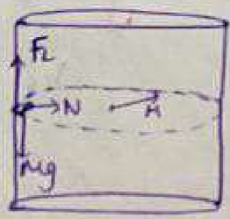
$$T = 2\pi\sqrt{\frac{L}{g}}$$



$$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Death Well or Rotor

to save biker



$F_L \geq mg$

$\mu N \geq mg$

Here N act as centripetal force.

$$N = F_c = \frac{mv^2}{r}$$

$$\frac{\mu \cancel{m} v^2}{r} \geq \cancel{m} g$$

$$\boxed{V = \sqrt{\frac{gr}{\mu}}} \qquad V_{min} = \sqrt{\frac{gr}{\mu}}$$

or

$$\mu \cancel{m} \omega^2 r \geq \cancel{m} g$$

Q→ Neet 2019

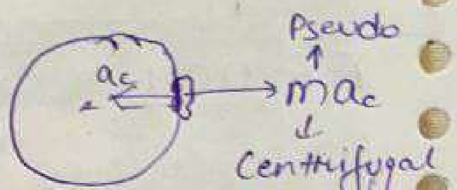
Note: Centripetal force is $\boxed{\text{NOT}}$ a new kind of Force. Some or other force act as centripetal force.

$$F_G = F_c = \frac{Gm_1m_2}{r^2}$$

Centrifugal Force

→ It is Pseudo force i.e (Imaginary force)

→ when observer is in rotating frame (accelerated frame) then we apply centrifugal force on the system

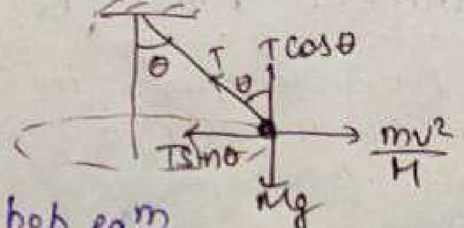


$F_{centrifugal} = m\omega^2 r$ = outward

System eq^m

When observer is in accelerated frame.

Conical pendulum



bob eq^m

$$T\cos\theta = mg$$

$$T\sin\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{gR}$$

$$\boxed{V = \sqrt{gR\tan\theta}}$$

Bending of Cyclist

$$\boxed{\tan\theta = \frac{v^2}{gR}}$$

(12) Turning of a Car :-

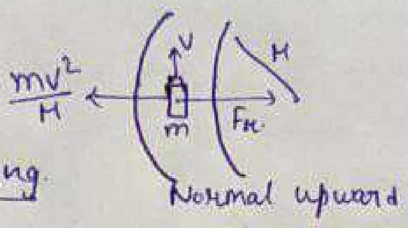
- In level Road (friction): $\boxed{V_{max} \leq \sqrt{\mu g r}}$
- Banking (Normal Rxn): $\boxed{V = \sqrt{gR\tan\theta}}$
- Banking + friction:

$$\boxed{V_{max} = \sqrt{\frac{g r(\tan\theta + \mu)}{1 - \mu\tan\theta}}}$$

$$\boxed{V_{min} = \sqrt{\frac{g r(\tan\theta - \mu)}{1 + \mu\tan\theta}}}$$

(I) Turning of Car on level Road

N.I.F.O.R.



For no skiding

$$\frac{mv^2}{r} \leq F_{lim}$$

$$\frac{mv^2}{r} \leq \mu N$$

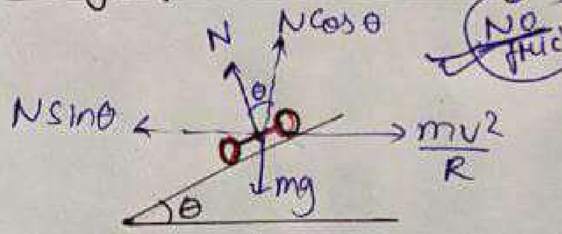
$$\boxed{V_{max} \leq \sqrt{\mu g r}}$$

In Raining $\mu \downarrow\downarrow \rightarrow F_{lim} \downarrow\downarrow$.

$V > V_{max} \rightarrow$ Skid.

(II) Turning of Car on banked Road :-

No fricn



$N\cos\theta = mg \quad | \quad N\sin\theta = \frac{mv^2}{r}$

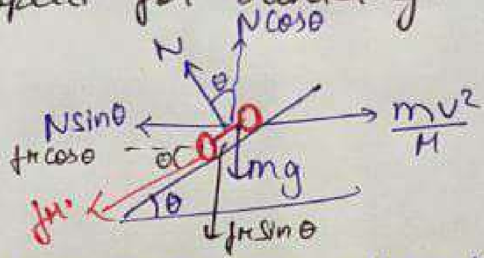
$$\boxed{\tan\theta = \frac{v^2}{g r}}$$

angle of banking.

$$\boxed{V = \sqrt{g r\tan\theta}}$$

(III) Banking with Friction

Case I → If speed of vehicle is greater than the designed speed for banking.



$V\uparrow\uparrow \rightarrow \frac{mv^2}{r}\uparrow\uparrow$, then $N\sin\theta$ will can't able to balance $\frac{mv^2}{r}$ then frictional force will come in down ward dirn.

$$N\cos\theta = mg + f_r\sin\theta$$

$$N\cos\theta = mg + \mu N\sin\theta$$

$$N\cos\theta - \mu N\sin\theta = mg \quad \text{(I)}$$

$$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{r} \quad \text{(II)}$$

(II) ÷ (I)

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{V_{max}^2}{g r}$$

divide by $\cos\theta$.

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{V_{max}^2}{g r}$$

$$\boxed{V_{max} = \sqrt{\frac{g r(\tan\theta + \mu)}{1 - \mu\tan\theta}}}$$

Case II :- $V < V_{max}$

fr. force upward dirn mai lagega.